Leap Frog

ACMNA296

Answers & Teacher Notes

7 8 **9** 10 11 12

TI-Nspire | Investigation | Student | 50 min

Objective

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations.

Introduction

With the exception of the African Clawed Frog, frogs can only jump forward. Frogs however can also creep forward, that is move forward without jumping, and so it is for this classic mathematics problem. Suppose there are six frogs sitting on some lilypads. Three frogs face toward the right, three to the left with a single vacant lilypad separating them all. In this problem the frogs on the left want to move to the right and visa versa. Is it possible given the restrictions on the frog's movements? How many moves would it take? What if more frogs come along?

The aim of this problem solving task is to:

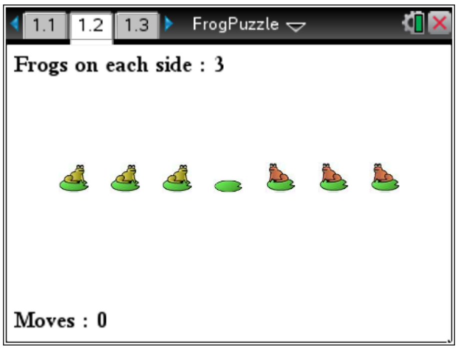
- Solve the 6 frog problem and record the number of moves. (3 frogs on each side)
- Determine a relationship between the number frogs and moves when more or less frogs exist. (Same number of frogs on each side)
- Determine a relationship for the general case with various quantities of frogs on each side.

Equipment

- TI-nspire calculator or computer software
- TI-Nspire FrogPuzzle file (TNS)

Instructions – Problem 1

Load the TI-Nspire **FrogPuzzle.tns** file onto the calculator.



Open the Frog Puzzle file and read the instructions. Navigate to page 1.2 and set up the problem with 3 frogs on each side.

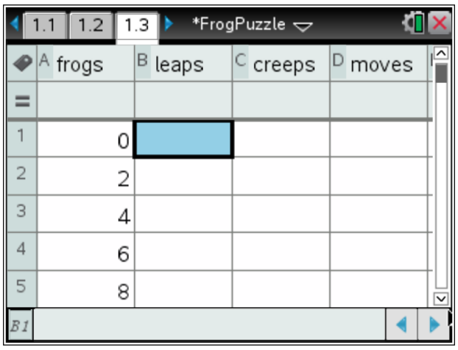
Use the [**menu**] key to change the number of frogs or start the problem over.

Use the mouse to click on a frog and make it move.

Pages that contain the puzzle have a different menu.

Solve the problem with three frogs on each side, six frogs in total. If you click on a frog and it doesn't move, it most likely means it is an invalid move.

Record the number of moves used in the appropriate cell of the spreadsheet on page 1.3.



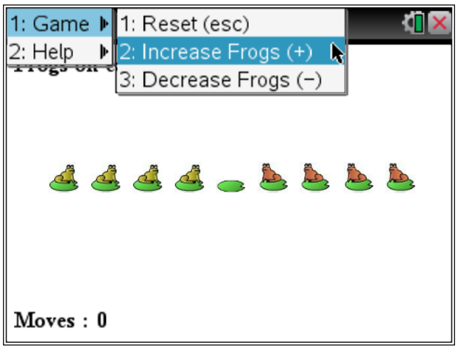
Question: 1.

How many moves does it take to solve the six frog problem? Answer: 15

Use the [**menu**] key to change the number of frogs, same quantity on each side.

Solve the 2, 4 and 8 frog problems.

Don't solve the 10 frog problem just yet!

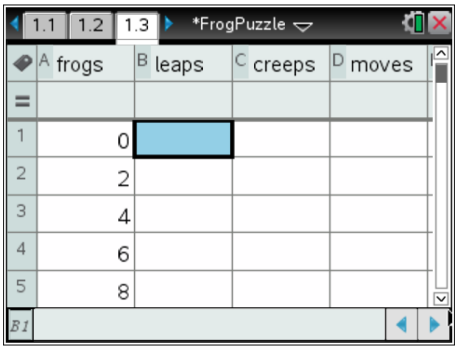


Question: 2.

Record the number of moves to solve the 2, 4 and 8 frog problems in the corresponding spreadsheet cells. Answers: 3, 8 and 24 respectively.

A rule exists that relates the number of moves required to solve the problem and the number of frogs. The rule is not obvious, to help determine the rule the number of 'leaps' and 'creeps' will be recorded.

Start the 2, 4 and 6 problems again, this time count and record the number of 'creeps' for each problem in the spreadsheet.



Question: 3.

Record the number of creeps to solve the 2, 4 and 6 frog problems in the corresponding spreadsheet cells. Comment on the relationship between creeps and frogs.

Answers: 2, 4, 6 respectively.

Relationship between number of frogs and number of creeps is linear, y = x.

Question: 4.

Use the number of moves and creeps to determine and record the number of leaps for the 2, 4 and 6 frog problems. Comment on the relationship between leaps and frogs.

Answers: 2 frogs => 3 (moves) – 2 (creeps) = 1 (leaps).

4 frogs => 8 (moves) – 4 (creeps) = 4 (leaps).

6 frogs => 15(moves) – 6 (creeps) = 9 (leaps). … leaps = $(\text{frogs}/2)^2$

Question: 5.

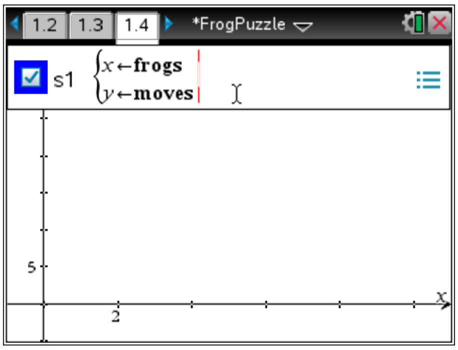
Use the patterns you have discovered to determine the number of leaps for the 8 frog problem.
General Rule: Moves – Creeps = Leaps. Therefore 24 – 8 = 16 leaps, which aligns with previous relationship: leaps = $(\text{frogs}/2)^2$

Graphing the relationship

Navigate to page 1.4 and set up a scatter plot to graph the data:

x axis = frogs (independent variable)

y axis = moves (dependent variable)



Question: 6.

Describe the relationship between the number of frogs and the number of moves. Include reference to what happens to the total number of moves each time two additional frogs are added to the problem.
Answer / Comment: Answers will vary but should reference non-linear either directly or as a non-constant difference, such as 'does not go up by the same amount each time'.
Teacher – A demonstration of 'difference' tables in the spreadsheet is a worthy discussion and inclusion at this point.

Defining rules or functions

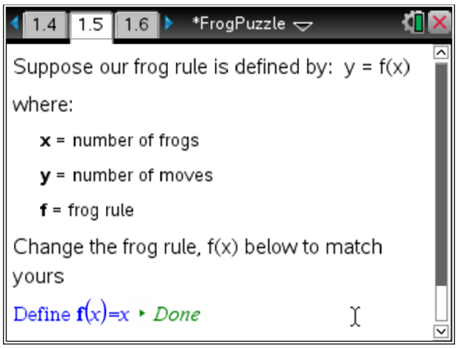
Defining a function makes it much easier to calculate, solve and graph a rule. On the calculator the function (rule) may be called up in any of the applications within the same problem.

Navigate to the notes application on page 1.5.

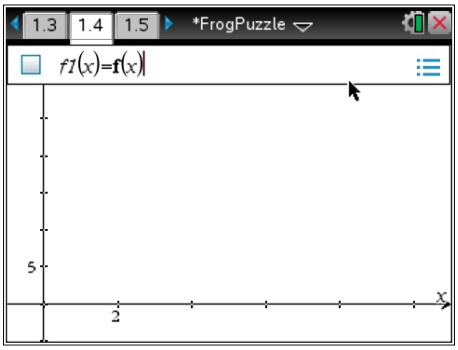
A rule is defined at the base of this page; this rule needs to be changed to represent the actual relationship.

Define $f(x) =$

Write your rule, in terms of x after the equals sign and press [**enter**] to confirm.

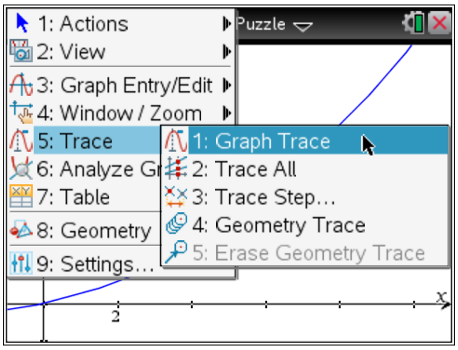


Return to the graph application, press the [**menu**] key, change the **graph entry / edit** to a **function** and type in your frog rule: $f(x)$



Press Ctrl + Up (arrow) to view the entire document. This is similar to the Power Point slide view. Navigate to the appropriate page and press [**enter**].

Press the [**menu**] key and use the **Trace** - **Graph** option to check the values generated by the rule against the data.



Question: 7.

State the rule that relates the number of frogs to the number of moves anc compare the values generated by the rule with those obtained practically. Test other values and comment on the limitations of the rule.

Answer: Results are the same, students should include sample calculations, but the rule is limited to positive even numbers of frogs.

Question: 8.

Use your rule to predict how many moves would be required to solve the 10 frog problem.

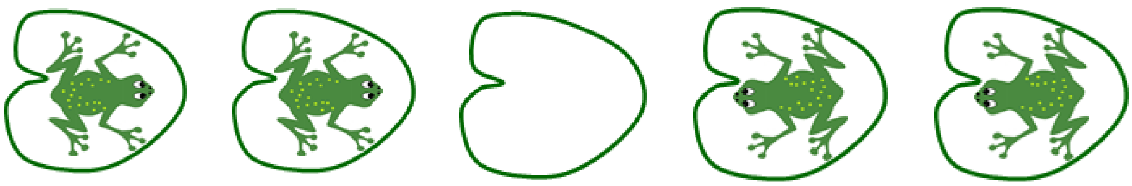
Answer: f(10) = 35

Question: 9.

Fred Dough spent almost half an hour making 195 moves solving the frog problem. Assuming he didn't make any mistakes and there were the same number of frogs on each side, how many frogs were in Fred's problem? Answer: Solve(f(x)=195,x) x = 26 (reject x = -30)

Understanding the Rule

To help understand the rule it is useful to break the problem back down to 'leaps' and 'creeps'. Answer the following questions to help explain and understand the formulation of the rule and therefore develop a rule when there are different quantities of frogs on either side of the puzzle.



Frog:	**1**	**2**		**3**	**4**
Position:	**A**	**B**	**C**	**D**	**E**

Question: 10.

Frog 2 ultimately needs to reach position E. How many positions does frog 2 need to move?

Answer: 3 positions

Question: 11.

How many positions does each frog need to move?

Answer: 3 positions for all frogs. [For n frogs there would be n + 1 positions]

Question: 12.

To get to the final position, frog 2 must get past frog 3 and 4, the only way for the frogs to move past one another is to ‘leap’. How many leaps must take place for frog 2 to get past frog 3 and 4?

Answer: 2 leaps. Doesn’t matter which frog leaps which, but 2 leaps must occur.

Question: 13.

How many leaps must take place for frog 1 to get past frog 3 and 4?

Answer: 2 leaps (as above.) [For n frogs on each side, there would be n leaps]

Question: 14.

How many leaps must take place to solve the problem?

Answer: Total of four leaps, the two frogs on the left must pass the two frogs on the right.

Comment: For n frogs on each side, n^2 leaps, or x frogs altogether $(x/2)^2$ leaps.

Question: 15.

Each time a leap takes place a frog moves 2 positions. Combine this information with the total number of leaps and total number of ‘positions’ to calculate the total number of creeps.

Answer: If there are 4 frogs, 2 on each side …

Leaps = 2 x 2 = 4 But this means 4 x 2 = 8 positions are gained.

Positions: 2 x 3 (LHS) + 2 x 3 (RHS) = 12

Creeps = 12 – 8 = 4

Question: 16.

Use your answers from questions 10 to 15 to fully explain the rule that relates the number of frogs to the number of moves, including any limitations on the rule.

Answer: Leaps = $(n/2)^2$ Positions gained: $2 \times (n/2)^2$ or $n^2 / 2$

Positions = $(n) \times (n/2 + 1) = n^2/2 + n$ (Each frog needs to move n/2 + 1 positions)

Creeps = $(n^2/2 + n) – n^2 /2 = n$

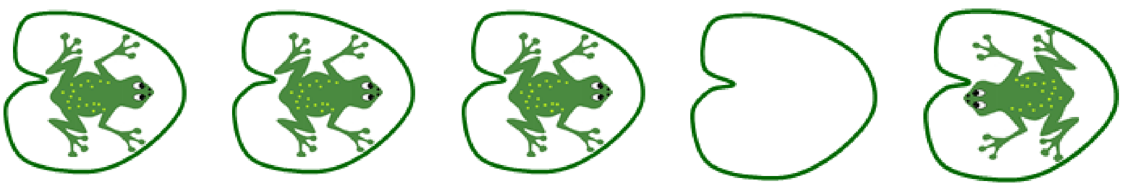
Rule: Moves = Leaps + Creeps = $(n/2)^2 + n$

Generalising the Rule - Extension

So far only equal quantities of frogs on both sides have been considered. In this problem the number of frogs on each side can vary. For the 4 frog problem could be:

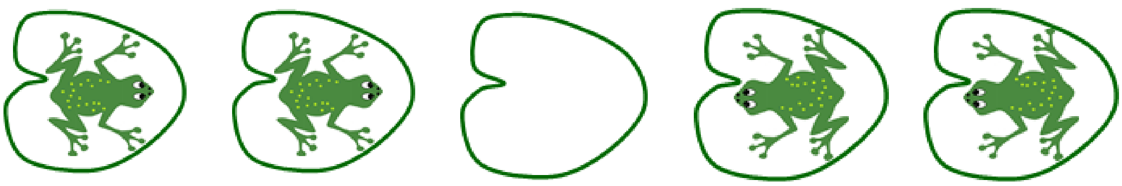
Prob. 4.1

(3, 1)



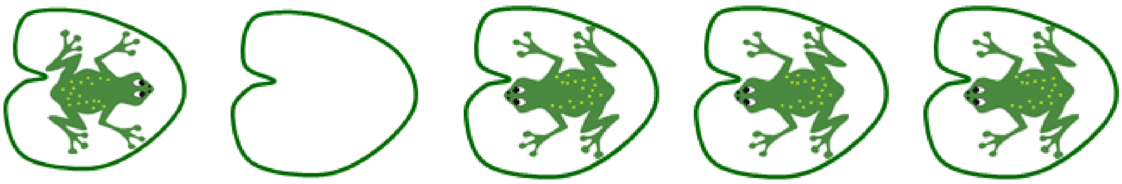
Prob. 4.2

(2, 2)



Prob. 4.3

(1, 3)



There are no options for 0 frogs on either side of the puzzle, leaving only three ways to organise the frogs. Notice that problem 4.1 and 4.3 would result in the same number of moves and problem 4.2 has been solved previously.

Question: 17.

How many moves are required to solve problem 4.1? Answer: 7 moves

Note: Go to page 2.1 to solve the problem digitally and use the [**menu**] to set up the problem.

Question: 18.

How many set up options are available for the 6 frog problem?

Answer: (5,1) (4, 2) & (3,3). The reverse set up (2, 4) and (1, 5) also exist but result in the same number of moves.

Question: 19.

For each of the 6 frog problems determine the number of moves.

Answer: (5, 1) = 11 moves Same as (1, 5)
(4, 2) = 14 moves Same as (2, 4))
(3, 3) = 15 moves

Question: 20.

Use the same logic applied to the original problem to find a rule where the number of frogs on each side can be changed independently.

Let m = Number of frogs on the left

and n = Number of frogs on the right.

Check your answer using question 17 and 19 and compare this new rule to the original.

Answer: f(n,m) = n x m + m + n

[Q.17]: f(1,4) = 7, [Q.19]: f(1,5) = 11, f(4,2) = 14, f(3,3) = 15

Effectively the same as the original rule. ie: Let x = 2n where n is the number no each side.

Question: 21.

When Fred Dough finished his puzzle he gave the frogs to Carmello Corewalla who used them all to solve the frog problem with different numbers of frogs on each side. Carmello solved the problem efficiently in 159 moves. How were the frogs distributed in Carmello's problem?

Answer: Fred Dough used 26 frogs. Given Carmello used the same amount there would be *n* frogs on one side and 26 – n on the other. Using the rule from the previous problem.

Number of moves = n x (26 – n) + (26 – n) + n = 159.

Solve($-n^2 + 26n + 26 = 159$, n) results in n = 7 or n = 19 both these answers are correct.

Note the symmetry of the result. LHS = 7 frogs means RHS = 19 frogs and visa versa.